Exercise 25

If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a+1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a+h)$.

Solution

$$f(2) = 3(2)^2 - (2) + 2 = 3(4) = 12$$

$$f(-2) = 3(-2)^2 - (-2) + 2 = 3(4) + 2 + 2 = 16$$

$$f(a) = 3a^2 - a + 2$$

$$f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2$$

$$\begin{aligned} f(a+1) &= 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a - 1 + 2 = 3a^2 + 6a + 3 - a + 1 \\ &= 3a^2 + 5a + 4 \end{aligned}$$

$$2f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4$$

$$f(2a) = 3(2a)^2 - (2a) + 2 = 3(4a^2) - 2a + 2 = 12a^2 - 2a + 2$$

$$f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3a^4 - a^2 + 2$$

$$\begin{aligned} [f(a)]^2 &= (3a^2 - a + 2)(3a^2 - a + 2) = 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4 \\ &= 9a^4 - 6a^3 + 13a^2 - 4a + 4 \end{aligned}$$

$$f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 + 6ah + 3h^2 - a - h + 2$$